Localization of light in a lamellar structure with left-handed medium : the light wheel

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Abstract

The contra-directional coupling between a left-handed monomode waveguide and a right-handed monomode waveguide is rigorously studied using a complex plane analysis. Light is shown to rotate in this lamellar structure forming a very exotic mode which we have called a light wheel. The light wheel can be excited using evanescent coupling or by placing sources in one of the waveguides. This structure can thus be seen as a new type of cavity. It is a way to suppress the guided mode of a dielectric slab.

Left-Handed Materials (LHM) present simultaneously negative permittivity and permeability. Such materials were a pure theoretical oddity[1] until the recent experimental demonstration of the negative refraction[2]. Left-handed materials can be made using metamaterials, *i.e.* periodical structures with a period smaller than the wavelength, which behave as a homogeneous medium. Consequently to these works the LHM have aroused huge interest. It clearly appeared that LHM had hardly been studied although they could lead to structures presenting highly nonconventional behaviours : they seemingly allow to overcome the Rayleigh limit[3] and could even lead to invisibility[4]. In a large number of situations, LHM behave the opposite way of conventional (right-handed) dielectric materials. This is of course the case for refraction (so that LHM are sometimes said to present a negative index), but for the Goos-Hänchen effect as well[5, 6]. Generally, lamellar structures made with left-handed materials present exotic properties. It has been shown for instance that a LHM slab could support backward or forward guided modes[10] and backward or forward leaky modes [7, 8, 9].

In this paper we present the study of the contra-directional coupling between a guided mode supported by a dielectric slab and a backward mode supported by a left-handed slab[10]. We show that this structure behaves as an edge-less cavity because light rotates inside the structure, and that the modes involved in this behaviour can be excited either using a prism coupler or by a source in one of the waveguides - leading to the formation of a "light wheel". Such a cavity would be obtained without introducing any type of defect, as in the case of photonic crystals.

The considered structure is presented figure 1 : two slabs are surrounded by air and separated by a distance h. The upper slab, whose thickness is called h_1, presents a relative permittivity of ϵ_1 and a relative permeability μ_1 while the lower slab, whose thickness is h_2, is characterized by ϵ_2 and μ_2. This rather simple structure allows analytical calculations and a rigorous analysis of the contra-directional coupling using complex plane analysis - which is almost never done.

We will search for solutions which present a harmonic time dependence and which do not depend on y - just like the geometry of the problem. With these assumptions, the solutions can be either TE polarized (*i.e.* the electric field is polarized along the y axis) or TM polarized (the magnetic field being polarized along the y axis). In this work, we will only consider the TE case but the same study can be carried out for the TM case, leading to the same conclusions. We will hence seek a solution for which $E_y(x,z,t) = E(z)\,\exp(i(\alpha\,x - \omega\,t))$ with $\omega = k_0\,c = \frac{2\pi}{\lambda}\,c$ where λ is the wavelength in the vacuum. In the following, we will consider that λ is the distance unity (*i.e.* we take $\lambda = 1$).

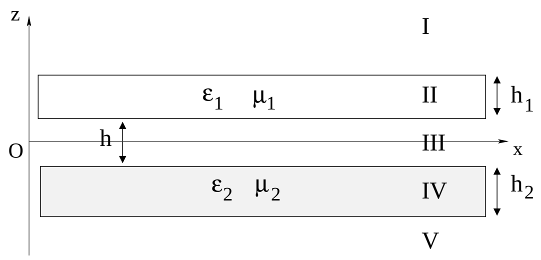

Figure 1: The two waveguides are surrounded by air and separated by a thickness h. The $z = 0$ plane is placed in the middle of the air layer (region III). The different regions are labelled using roman numerals.

The function $E(z)$ in a medium j characterized by ϵ_j and μ_j is in general a linear combination of two exponential terms $\exp(\pm i\,\gamma\,z)$

with $\gamma_j^2 + \alpha^2 = \epsilon_j \, \mu_j \, k_0^2$. Since we are only interested in guided modes, we will consider that $\alpha > k_0$ so that the electric field is decaying exponentially in the air (we will note $\gamma_0 = \sqrt{\alpha^2 - k_0^2}$). Above and under the waveguides (in regions I and V) only one decaying exponential term remains. In this case, the continuity relations at an interface between two different media can be written as a homogeneous system of equations. A non null solution can thus be found only if the determinant of the system is null, which can be written

$$x_1 \, F_1 + x_2 \, F_2 + (1 + x_1 \, x_2 \, F_1 \, F_2) \, \tanh(\gamma_0 \, h) = 0 \tag{1}$$

where $F_j = \frac{1 - x_j \, \tan(\gamma_j \, h_j)}{x_j + \tan(\gamma_j \, h_j)}$ and $x_j = \frac{\gamma_j}{\mu_j \, \gamma_0}$ whatever the sign of ϵ_j or μ_j. Relation (1) is the dispersion relation because each zero of the left part is a mode supported by the structure.

The coupling of the two waveguides appears when the upper and the lower waveguides, taken separately, both support a guided mode for the same constant propagation, called α_0. First we have chosen a small thickness for the right-handed slab ($\epsilon_1 = 3$, $\mu_1 = 1$, $h_1 = 0.2 \, \lambda$) so that is behaves as a monomode waveguide and we have computed α_0. If the two waveguides are identical (both right-handed and with the same thickness) the structure supports two propagating modes with two close and real propagation constants. The excitation of these two modes gives raise to an oscillation of the energy between the two guides as shown figure2.

When the lower waveguide is left-handed (with $\epsilon_2 = -3$, $\mu_2 = -1$), its thickness has to be chosen carefully to ensure a perfect coupling with the above slab. Using the dispersion relation of the left-handed waveguide[10] we have chosen the thickness of the LHM slab ($h_2 = .7146 \, \lambda$) so that (i) it supports no fundamental mode (which would otherwise propagate forward) and (ii) the only supported mode is more localized in the LHM and hence propagates backward.

Figure 2: The two identical coupled waveguides of height $0.2 \, \lambda$ are excited using evanescent coupling : an incident beam with an incidence angle of 36, 9 in a medium with $\epsilon = 5$ and $\mu = 1$ is used. The energy oscillates between the two guides and leaks out when it is in the upper guide, so that it decays exponentially.

With the above parameters the dispersion relation is verified only for complex values of α : the two different solutions have then the same real part but opposite imaginary parts (*i.e.* conjugate propagation constants). The solution in the complex plane are shown figure 3 for different values of h.

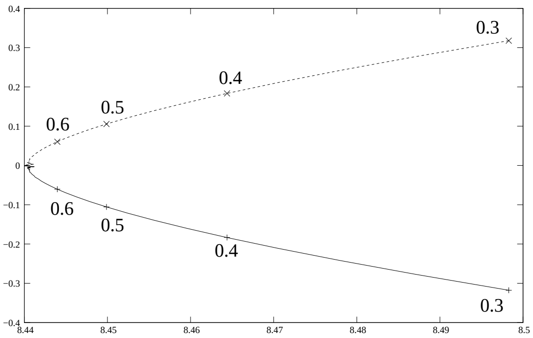

Figure 3: Solutions of the dispersion relation in the α complex plane for different values of h, which is given for some points as a fraction of the wavelength. The waveguides are characterized by $h_1 = 0.2 \, \lambda$, $\epsilon_1 = 3$, $\mu_1 = 1$, $\epsilon_2 = -3$, $\mu_2 = -1$ and $h_2 = .7146 \, \lambda$. The units correspond to the choice $\lambda = 1$.

Let us now study the characteristics of

the solutions, which are shown figure 4. Figure 4 shows the modulus, the phase and the time-averaged Poynting vector along the x direction for the solution corresponding to the propagation constant with a positive imaginary part with $h = 0.5$. The solution is an hybrid mode, since the field is important in both waveguides (see figure 4 (a)). The field in the right-handed medium and the field in the left-handed medium are un phase quadrature : there is a phase difference of $\frac{\pi}{2}$ in between as shown figure 4 (b). Since the propagation constant of the other solution is the conjugate of the one represented figure 4, then the solution itself is the conjugate of the first one. The only difference between the two solutions is hence the phase, so that in both cases there is a phase quadrature between the two guides. This is very different from what happens with co-directional coupled waveguides in which the two guides are in-phase or out-of-phase, depending on the considered mode (see figure 2).

The time-averaged Poynting vector along the x direction is shown figure 4 (c). As expected, it is negative in the LHM and positive elsewhere. It is very interesting to note that *the overall power flux of each mode is found to be null.* This means that all the energy that is sent in one direction by one of the waveguides comes back using the other one. This helps to understand the nature of the modes : although they have an important real part of the propagation constant along the x axis, they can be considered as evanescent modes because they do not convey any energy. It is then difficult to define a *propagation* direction, but the imaginary part of the propagation constant suggests that the modes actually have a direction in which they *developp*. When the imaginary part is positive, the mode is decaying when x is increasing. It is thus expected to developp towards the right - otherwise it would diverge. This means that the other mode developps in the other direction. This is a discussion which is usually applied to leaky modes[9].

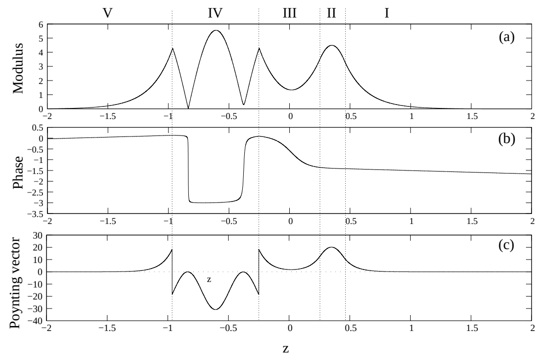

Figure 4: Characteristics of the solution with a propagation constant presenting a positive imaginary part for $h = 0.5\,\lambda$. Figure (a) shows the modulus of the field (which presents two zeros in the left-handed guide), figure (b) the phase, which is either null or equal to π in one guide and to $\frac{\pi}{2}$ in the other one, and figure (c) shows the Poynting vector along the x direction.

Let us just underline that these two modes, having the same real part of their propagation constant, cannot be excited separately. They can be excited using evanescent coupling, *i.e.* using an incident beam propagating in a high-index medium close to the waveguides. When the incidence angle of the beam is greater than the critical angle, the waveguided modes can be excited. Figure 5 shows such a situation. The field is clearly enhanced in the two waveguides, and the structure of the mode in the left-handed slab is particularly visible. The most striking feature is the fact that there is a dark zone just above the incident beam in the closest slab. It is produced by the superposition of the two modes - and the center of the beam is the only place where it may happen since the modes develop in different directions. Such a dark zone could be expected - after all, this is the case figure 2 - but in the lower waveguide. This is actually a characteristic of this particular coupling and it persists even when the two slabs are exchanged.

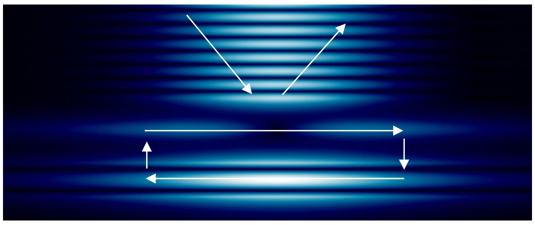

Figure 5: Excitation of the light wheel using an incident beam in a high-index material (with $\epsilon = 5$ and $\mu = 1$). The image represents the modulus of the E_y field in a domain which is only $4.5\,\lambda$ high and about $60\,\lambda$ large. The propagation direction of light is indicated by white arrows so that the rotation of light in the structure is made visible. The fringes above the structure are localized interferences of the incident and reflected beams. The profiles of the modes are in perfect agreement with figure 4. Notice the dark zone just under the incident beam. The incident angle of the beam is 36.9, the distance between the prism and the first waveguide is $0.5\,\lambda$.

Finally, when the two modes are excited they form what we called a *light wheel* : the light is heading to the right in the right-handed waveguide and to the left in the left-handed slab. On the right of the incident beam, the energy flows from the right-handed slab to the left-handed slab. This is the contrary on the left of the beam, so that light globally rotates inside the structure. In the case of an evanescent coupling, the reflected beam is distorted because of the excitation of the light wheel - but it is distorted symetrically if the coupling is good enough. Such a simple structure could then be used to perform beam reshaping[11].

The previous results suggest that despite its lamellar geometry the structure can be considered *as a cavity*. To ensure that an atom placed inside a waveguide is actually coupled with the light wheel we have computed the Green function of the problem. Figure 6 shows the Green function when a source is place in the middle of the dielectric slab, and the light wheel is clearly excited. More precisely, two contra-rotative light wheels are excited and interfere.

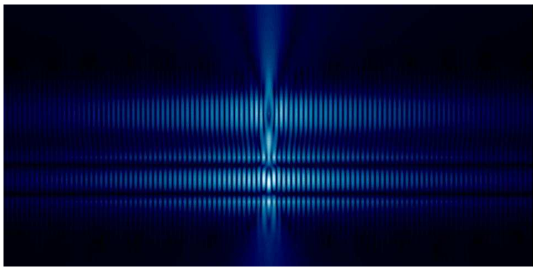

Figure 6: Modulus of the electric field when a punctual source is placed in the middle of the right-handed waveguide. Two contra-rotative light wheels are excited and they interfere which explains the interference fringes.

In conclusion, we have shown that the use of a left-handed medium in a lamellar structure could lead to a new type of phenomenon : the rotation of light which would then form a light wheel. This phenomenon could be used for beam reshaping, or even as a new type of cavity. It is an original way to suppress any propagative mode in a dielectric slab.

The properties of the structure are likely to be improved. Using Bragg mirrors could for instance enhance the quality factor of such a cavity by hindering light from leaking out of the structure. We believe that this study brings a new evidence that left-handed materials are renewing the physics of lamellar structures in photonics[12].

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